

# The masses of gauge fields in higher spin field theory on the bulk of $AdS_4$

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## ABSTRACT

A local gauge invariant interaction Lagrangian for two gauge fields of spin  $\ell$  and  $\ell - 2$  ( $\ell > 2$ ) and the scalar field is defined. It gives rise to one-loop corrections to the gauge field propagator. The loop function contains the Goldstone boson propagator for gauge symmetry breaking. The proportionality factor in front of this propagator is the mass squared of the gauge boson.

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# 1 Introduction

The *AdS/CFT* correspondence [1] opened a new era in understanding of the strong coupling regime of certain conformal boundary theories mapping them on the perturbation expansion of the weakly coupled string/supergravity theory on the bulk *AdS* space. In contrast to the most explored case of *AdS*<sub>5</sub>/*CFT*<sub>4</sub> dealing with the large  $N$  limit of  $SU(N)$   $\mathcal{N} = 4$  Super Yang-Mills theory, the *AdS*<sub>4</sub>/*CFT*<sub>3</sub> correspondence of the critical  $O(N)$  sigma model [2] works in the regime of small t'Hooft coupling and turns our attention from supersymmetric string/gravity theories to the theory of massless gauge higher even spins [3]. This case is interesting also in view of the unique properties of the renormalization group flow connecting the free field unstable point of the boundary  $O(N)$  vector model with the stable critical interacting conformal point in the large  $-N$  limit by the deformation with the double trace marginal operator and Legendre transformation. This flow can be explained from the bulk side using the same higher spin theory ( $HS(4)$ ) and different boundary conditions for the quantized scalar field [4]. Note that in the second and nontrivial conformal point of  $d = 3$  sigma model all higher spin currents except the energy-momentum tensor (spin two) are conserved only in the large  $N$  limit and their divergence is of first order in  $\frac{1}{N}$ . On the bulk side this must correspond to a certain mass generation mechanism on the one loop level (again order of  $\frac{1}{N}$ ) of the interacting  $HS(4)$  gauge theory.

In this article we explore this mechanism of the one-loop mass generation for massless gauge  $HS$  fields from the bulk side proposed in [5] and compare it with the corresponding boundary sigma model derivation of one of the authors [6]. For this we construct the only one possible  $h^{(\ell)}h^{(\ell-2)}\sigma$  local interaction of fields with spin  $\ell$ ,  $\ell - 2$  and zero in the way similar to our previous investigation of the  $h^{(\ell)}\sigma\sigma$  vertex [7, 8]. The coupling constant  $g_\ell$  of the vertex  $h^{(\ell)}h^{(\ell-2)}\sigma$  remains undetermined, in particular we do not relate it to the universal coupling constant of  $HS(4)$ . Thus the validity of *AdS/CFT* correspondence on the one-loop level remains open.

## 2 Equations of motion and physical states

In this article we operate with the following objects: The conformally coupled scalar field satisfying the usual equation of motion in  $AdS_{d+1}$  space<sup>†</sup>

$$\square\sigma(z) = \frac{(d+1)(d-1)}{4}\sigma(z), \quad (1)$$

general spin  $\ell$  symmetric and double traceless conserved currents  $J_{\mu_1\mu_2\dots\mu_\ell}^{(\ell)}(z)$ , and again double traceless [9] spin  $\ell$  gauge fields  $h_{\mu_1\mu_2\dots\mu_\ell}^{(\ell)}(z)$ . For shortening the notation and calculation we contract all symmetric tensors with the  $\ell$ -fold tensor product of a vector  $a^\mu$ . In this notation Fronsdal's equation of motion [9] for the spin  $\ell$  field is

$$\begin{aligned} \mathcal{F}(h^{(\ell)}(z; a)) &= \square h^{(\ell)}(z; a) - \ell(a\nabla)\tilde{h}^{(\ell-1)}(z; a) + \frac{\ell(\ell-1)}{2}(a\nabla)^2\bar{h}^{(\ell-2)}(z; a) \\ &+ \frac{\ell^2 + \ell(d-5) - 2(d-2)}{L^2}h^{(\ell)} + \frac{\ell(\ell-1)}{L^2}a^2\bar{h}^{(\ell-2)}(z; a) = 0, \end{aligned} \quad (3)$$

$$\square_a\square_a h^{(\ell)} = 0, \quad \square_a = \frac{\partial^2}{\partial a^\mu \partial a_\mu}, \quad (4)$$

where we introduced notations for the trace and divergence

$$\bar{h}^{(\ell-2)} = \frac{1}{\ell(\ell-1)}\square_a h^{(\ell)}, \quad \tilde{h}^{(\ell-1)} = \frac{1}{\ell}\nabla^\mu \frac{\partial}{\partial a^\mu} h^{(\ell)}. \quad (5)$$

The basic property of this equation is higher spin gauge invariance with the traceless parameter  $\epsilon^{(\ell-1)}(z; a)$ ,

$$\delta h^{(\ell)}(z; a) = (a\nabla)\epsilon^{(\ell-1)}(z; a), \quad \square_a \epsilon^{(\ell-1)}(z; a) = 0, \quad \delta \mathcal{F}(h^{(\ell)}(z; a)) = 0. \quad (6)$$

The equation (3) is simplified in the so-called de Donder gauge

$$\tilde{h}^{(\ell-1)} = \frac{\ell-1}{2}(a\nabla)\bar{h}^{(\ell-2)}, \quad (7)$$

$$\begin{aligned} \mathcal{F}^{dD \text{ gauge}}(h^{(\ell)}) &= \square h^{(\ell)} + \frac{\ell^2 + \ell(d-5) - 2(d-2)}{L^2}h^{(\ell)} \\ &+ \frac{\ell(\ell-1)}{L^2}a^2\bar{h}^{(\ell-2)} = 0. \end{aligned} \quad (8)$$

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<sup>†</sup>We will use  $AdS_{d+1}$  conformal flat metric, curvature and covariant derivatives commutation rules of the type

$$\begin{aligned} ds^2 &= g_{\mu\nu}dz^\mu dz^\nu = \frac{L^2}{(z^0)^2}\eta_{\mu\nu}dz^\mu dz^\nu, \quad \eta_{z^0 z^0} = -1, \sqrt{-g} = \frac{1}{(z^0)^{d+1}}, \\ [\nabla_\mu, \nabla_\nu]V_\lambda^\rho &= R_{\mu\nu\sigma}{}^\rho V_\lambda^\sigma - R_{\mu\nu\lambda}{}^\sigma V_\sigma^\rho, \\ R_{\mu\nu\lambda}{}^\rho &= -\frac{1}{(z^0)^2}(\eta_{\mu\lambda}\delta_\nu^\rho - \eta_{\nu\lambda}\delta_\mu^\rho) = -\frac{1}{L^2}(g_{\mu\lambda}\delta_\nu^\rho - g_{\nu\lambda}\delta_\mu^\rho), \\ R_{\mu\nu} &= -\frac{d}{(z^0)^2}\eta_{\mu\nu} = -\frac{d}{L^2}g_{\mu\nu}, \quad R = -\frac{d(d+1)}{L^2}. \end{aligned}$$

It was shown (see for example [10]) that in the de Donder gauge the residual gauge symmetry leads to the tracelessness of the on-shell fields. So we can define our massless physical spin  $\ell$  modes as traceless and transverse symmetric tensor fields satisfying the equation (7)

$$\left[ \square + \frac{\ell^2 + \ell(d-5) - 2(d-2)}{L^2} \right] h^{(\ell)} = 0, \quad (9)$$

$$\bar{h}^{(\ell-2)} = \frac{1}{\ell(\ell-1)} \square_a h^{(\ell)} = 0, \quad (10)$$

$$\tilde{h}^{(\ell-1)} = \frac{1}{\ell} \nabla^\mu \frac{\partial}{\partial a^\mu} h^{(\ell)} = 0. \quad (11)$$

Note that equation (9) for  $\ell = 0$  coincides with the equation for the conformal scalar (1) only for  $d = 3$ .

In a similar way we can describe the massive higher spin modes using the following set of constraints on the general symmetric tensor field  $\phi^{(\ell)}(z, a)$  [11]

$$\left[ \square + \frac{\ell^2 + \ell(d-5) - 2(d-2)}{L^2} + m^2 \right] \phi^{(\ell)} = 0, \quad (12)$$

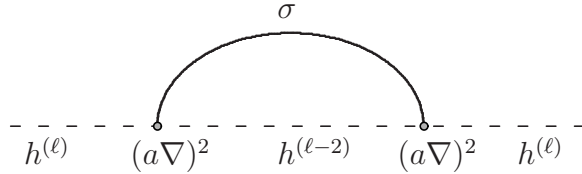
$$\bar{\phi}^{(\ell-2)} = \tilde{\phi}^{(\ell-1)} = 0, \quad (13)$$

$$L^2 m^2 = \Delta(\Delta - d) - (\ell - 2)(\ell + d - 2), \quad (14)$$

where  $\Delta$  is the conformal weight (dimension) of the corresponding massive (in means of *AdS* field) representation of the  $SO(d, 2)$  isometry group. This general representation (12)-(14) with two independent quantum numbers  $[\Delta, \ell]$  under the maximal compact subgroup  $SO(d) \times SO(2)$  goes, after imposing a shortening condition  $\Delta = \ell + d - 2$ , to the massless higher spin case (9)-(11) with the following decomposition [6, 12, 5]

$$\lim_{\Delta \rightarrow \ell + d - 2} [\Delta, \ell] = [\ell + d - 2, \ell] \oplus [\ell + d - 1, \ell - 1]. \quad (15)$$

The additional massive representation  $[\ell + d - 1, \ell - 1]$  is the Goldstone field. Reading this decomposition from the opposite side, we can interpret it as swallowing of the massive spin  $\ell - 1$  Goldstone field by the massless spin  $\ell$  field with generation of mass for the latter one. This mass generation can be achieved by the following one-loop diagram, [5]



where the  $h^{(\ell)} h^{(\ell-2)} \sigma$  coupling should include two derivatives for producing the Goldstone representation in the transverse traceless part of this loop. This coupling we will construct in the next section.

### 3 The $h^{(\ell)}h^{(\ell-2)}\sigma$ interaction

In this section we present the interaction Lagrangian responsible for the one loop mass generation mechanism for HS fields that is fixed in a unique way by the general concept of *linearized gauge invariance* (6). We want to construct a gauge invariant interaction between two double traceless HS fields  $h^{(\ell)}(z; a)$  and  $h^{(\ell-2)}(z; a)$  and one scalar field  $\sigma(z)$ . We will try to realize this interaction in the form of the spin  $\ell$  gauge field  $h^{(\ell)}$  times the current  $\Psi^{(\ell)}$  constructed from the spin  $\ell - 2$  field  $h^{(\ell-2)}$ , the scalar field  $\sigma$  and *two* ( $AdS_{d+1}$ ) covariant derivatives  $\nabla$ .

$$S_{int}^{(\ell)} = \frac{1}{\ell} \int d^{d+1}z \sqrt{g} h^{(\ell)\mu_1 \dots \mu_\ell} \mathcal{J}_{\mu_1 \dots \mu_\ell}^{(\ell)}, \quad (16)$$

$$h_{\alpha\beta\mu_5 \dots \mu_\ell}^{(\ell)\alpha\beta} = 0 \quad , \quad \mathcal{J}_{\alpha\beta\mu_5 \dots \mu_\ell}^{(\ell)\alpha\beta} = 0, \quad (17)$$

$$\delta_0 h_{\mu_1 \dots \mu_\ell}^{(\ell)} = \partial_{(\mu_1} \epsilon_{\mu_2 \dots \mu_\ell)}, \quad \epsilon_{\alpha\mu_4 \dots \mu_\ell}^\alpha = 0. \quad (18)$$

From this point of view the task reduces to the construction of the current  $\mathcal{J}^{(\ell)}(h^{(\ell-2)}, \nabla, \nabla, \sigma)$  satisfying the conservation condition

$$[\nabla^{\mu_1} \mathcal{J}_{\mu_1 \mu_2 \dots \mu_\ell}^{(\ell)}]^{traceless} = 0. \quad (19)$$

The conservation condition looks a little bit different from the usual one due to the double-tracelessness of the gauge field and current and tracelessness of the corresponding gauge parameter. The Fronsdal field  $\mathcal{J}^{(\ell)}$  can be presented then as

$$\mathcal{J}^{(\ell)}(z; a) = J^{(\ell)}(z; a) + \frac{a^2}{2(d+2\ell-3)} \Theta^{(\ell-2)}(z; a), \quad (20)$$

$$Tr \mathcal{J}^{(\ell)}(z; a) = \square_a \mathcal{J}^{(\ell)}(z; a) = \Theta^{(\ell-2)}(z; a), \quad (21)$$

$$Tr J^{(\ell)}(z; a) = \square_a J^{(\ell)}(z; a) = 0, \quad (22)$$

$$Tr \Theta^{(\ell-2)}(z; a) = \square_a \Theta^{(\ell-2)}(z; a) = 0. \quad (23)$$

The conservation condition (19) in this representation is

$$\nabla^\mu \frac{\partial}{\partial a^\mu} \mathcal{J}^{(\ell)}(z; a) = \frac{a^2}{2(d+2\ell-5)} Tr \nabla^\mu \frac{\partial}{\partial a^\mu} \mathcal{J}^{(\ell)}(z; a), \quad (24)$$

or

$$\nabla^\mu \frac{\partial}{\partial a^\mu} J^{(\ell)}(z; a) + \frac{(a\nabla)\Theta^{(\ell-2)}(z; a)}{(d+2\ell-3)} = \frac{a^2 \nabla^\mu \frac{\partial}{\partial a^\mu} \Theta^{(\ell-2)}(z; a)}{(d+2\ell-5)(d+2\ell-3)}. \quad (25)$$

For solving this we will introduce the following ansatz for traceless spin  $\ell$  and  $\ell - 2$  currents

$$J^{(\ell)}(z; a) = \sum_{i=0}^3 A_i \left[ J_i^{(\ell)}(z; a) - \frac{a^2 \square_a J_i^{(\ell)}(z; a)}{2(d + 2\ell - 3)} \right] + \frac{a^2}{L^2} C h^{(\ell-2)}(z) \sigma(z) + O(a^4, \frac{1}{L^4}), \quad (26)$$

$$\Theta^{(\ell-2)}(z; a) = \sum_{p=1}^6 B_p \left[ \Theta_p^{(\ell-2)}(z; a) - \frac{a^2 \square_a \Theta_p^{(\ell-2)}(z; a)}{2(d + 2\ell - 7)} \right] + O(a^4, \frac{1}{L^4}), \quad (27)$$

where we introduced all possible monomials of corresponding order

$$J_1^{(\ell)} = (a\nabla)^2 h^{(\ell-2)} \sigma, \quad J_2^{(\ell)} = (a\nabla) h^{(\ell-2)} (a\nabla) \sigma, \quad J_3^{(\ell)} = h^{(\ell-2)} (a\nabla)^2 \sigma, \quad (28)$$

$$\Theta_1^{(\ell-2)} = \nabla_\mu h^{(\ell-2)} \nabla^\mu \sigma, \quad \Theta_2^{(\ell-2)} = h_{;\mu}^{(\ell-3)} (a\nabla) \nabla^\mu \sigma, \quad \Theta_3^{(\ell-2)} = (a\nabla) h_{;\mu}^{(\ell-3)} \nabla^\mu \sigma, \quad (29)$$

$$\Theta_4^{(\ell-2)} = (a\nabla)^2 \bar{h}^{(\ell-4)} \sigma, \quad \Theta_5^{(\ell-2)} = (a\nabla) \bar{h}^{(\ell-4)} (a\nabla) \sigma, \quad \Theta_6^{(\ell-2)} = \bar{h}^{(\ell-4)} (a\nabla)^2 \sigma. \quad (30)$$

Substituting this ansatz in the conservation condition (19) and neglecting the noncommutativeness of the covariant derivatives we will come to the set of liner equations for the coefficients  $A_i$  and  $B_p$  with the unique solution up to an overall normalization constant  $A$

$$A_1 = \frac{1}{2} A_2 = A_3 = A, \quad (31)$$

$$B_1 = -4(d + 2\ell - 4)A, \quad (32)$$

$$B_2 = B_3 = -(\ell - 2)(d + 2\ell - 7)A \quad (33)$$

$$B_4 = -(\ell - 2)(\ell - 3) \left( \frac{1}{2}(d + 1) + \ell - 5 \right) A \quad (34)$$

$$B_5 = -(\ell - 2)(\ell - 3) \left( \frac{1}{2}(d + 1) + \ell - 6 \right) A \quad (35)$$

$$B_6 = (\ell - 2)(\ell - 3)A. \quad (36)$$

From (31) we see that the leading term of our current  $\mathcal{J}^{(\ell)}$  is the double full derivative

$$\mathcal{J}^{(\ell)}(z; a) = A(a\nabla)^2 (h^{(\ell-2)}(z; a) \sigma(z)) + \text{traces}. \quad (37)$$

Now we can restore noncommutativeness of derivatives

$$\begin{aligned} \square_a (a\nabla)^2 (h^{(\ell-2)} \sigma) &= 4\Theta_1^{(\ell-2)} + 4(\ell - 2) \left( \Theta_2^{(\ell-2)} + \Theta_3^{(\ell-2)} \right) \\ &+ (\ell - 2)(\ell - 3) \left( 3\Theta_4^{(\ell-2)} + 4\Theta_5^{(\ell-2)} + \Theta_6^{(\ell-2)} \right) + \frac{g_1(\ell)}{L^2} h^{(\ell-2)} \sigma + O(a^2), \end{aligned} \quad (38)$$

$$\begin{aligned} \nabla^\mu \frac{\partial}{\partial a^\mu} (a\nabla)^2 (h^{(\ell-2)} \sigma) &= (a\nabla) \left[ 4\Theta_1^{(\ell-2)} + (\ell - 2) \left( \Theta_2^{(\ell-2)} + \Theta_3^{(\ell-2)} \right) \right. \\ &\left. + \frac{(\ell - 2)(\ell - 3)}{2} \left( \Theta_4^{(\ell-2)} + \Theta_5^{(\ell-2)} \right) \right] + \frac{g_2(\ell)}{L^2} (a\nabla) (h^{(\ell-2)} \sigma) + O(a^2), \end{aligned} \quad (39)$$

and calculate the curvature ( $\frac{1}{L^2}$ ) correction coefficient  $C$  in (26)

$$C = \frac{1}{2} \left( \frac{g_1(\ell)}{d+2\ell-3} - g_2(\ell) \right), \quad (40)$$

$$g_1(\ell) = \frac{(d+1)^2}{2} + 3d + 8\ell - 25, \quad g_2(\ell) = \frac{(d+1)^2}{2} + 5d + 8\ell - 33. \quad (41)$$

It is easy to see that if we try to construct a traceless and conserved current, we have to set all the coefficients  $B_p$  to zero and as a result will get zero for all  $A_i$ . So we deduce that contrary to the  $h^{(\ell)}\sigma\sigma$  case [8], the interaction of the type  $h^{(\ell)}h^{(\ell-2)}\sigma$  exists only in the Fronsdaal formulation with a double traceless current. At the end of this section it is worth to note that our interaction is unique because it is fixed by gauge invariance. Of course we considered only invariance with respect to transformation of only the highest spin  $\ell$  gauge field participating in the interaction, but we can assume on this level of consideration that variation of the scalar and spin  $\ell-2$  fields containing more derivatives of the parameter  $\epsilon$  (see [8]) will be compensated by variations of other unknown terms of the interacting Lagrangian.

## 4 Massive higher spin states and Goldstone boson

Here we consider the origin of the Goldstone boson for general massless spin  $\ell$  field. We can present this mechanism in two ways generalizing the consideration for the graviton of Ref. [12]. First of all we can try to extract the Goldstone field from the longitudinal part of the spin  $\ell$  physical mode represented by the set of equations (9)-(11). For this purpose we insert in the latter set the following ansatz

$$h^{(\ell)}(z; a) = (a\nabla)\phi^{(\ell-1)}(z; a). \quad (42)$$

Then using the relation:

$$[\nabla_\mu, (a\nabla)]\partial_{a\mu}\phi^{(\ell-1)} = \frac{(\ell-1)(\ell+d-2)}{L^2}\phi^{(\ell-1)} - \frac{(\ell-1)(\ell-2)a^2}{L^2}\bar{\phi}^{(\ell-3)} \quad (43)$$

we obtain the following equations for  $\phi^{(\ell-1)}(z; a)$

$$\left[ \square + \frac{(\ell-1)(\ell+d-2)}{L^2} \right] \phi^{(\ell-1)} = 0, \quad (44)$$

$$\bar{\phi}^{(\ell-3)} = \frac{1}{\ell(\ell-1)}\square_a\phi^{(\ell)} = 0, \quad (45)$$

$$\tilde{\phi}^{(\ell-2)} = \frac{1}{\ell}\nabla^\mu\frac{\partial}{\partial a^\mu}\phi^{(\ell-1)} = 0. \quad (46)$$

Comparing with (12) and (14) we deduce that our field  $\phi^{(\ell-1)}(z; a)$  is the massive spin  $\ell - 1$  representation of the isometry group with  $\Delta = \ell + d - 1$  and should describe the corresponding Goldstone mode.

The important point of this consideration is the following: For the traceless spin  $\ell - 1$  field  $\phi^{(\ell-1)}(a; z)$  the correct second order equation (44) for the massive states with  $\Delta = \ell + d - 1$  originated not only from the gauge fixed equation of motion (9) for the massless field  $h^{(\ell)} = (a\nabla)\phi^{(\ell-1)}$  but also from the gauge condition (11) using only the commutation rule (43). In other words we see that

$$\nabla^\mu \frac{\partial}{\partial a^\mu} (a\nabla)\phi^{(\ell-1)} = K_G^{-1}\phi^{(\ell-1)}, \quad (47)$$

where  $K_G^{-1} = \square + \frac{(\ell-1)(\ell+d-2)}{L^2}$  is the inverse propagator for the Goldstone mode  $[\ell + d - 1, \ell - 1]$ . This allows us to formulate a field theoretical realization of the representation decomposition formula (15) in a similar way as it was done in [12] for the graviton case.

Let us describe the massive spin  $\ell$  representation  $[\Delta, \ell]$  by the gauge invariant massless action  $S_{\mathcal{F}}^{GI}[h^{(\ell)}]$ , leading to the Fronsdal equation (3), perturbed by the mass term breaking gauge invariance. Neglecting the traces we can write this mass term in the usual way

$$S_m[h^{(\ell)}] = \frac{m^2}{2} \int d^{d+1}z \sqrt{-g} h^{(\ell)\mu_1 \dots \mu_\ell} h_{\mu_1 \dots \mu_\ell}^{(\ell)}, \quad (48)$$

$$L^2 m^2 = \Delta(\Delta - d) - (\ell - 2)(\ell + d - 2). \quad (49)$$

The action  $S_{\mathcal{F}}^{GI}[h^{(\ell)}] + S_m[h^{(\ell)}]$  is not gauge invariant and corresponds to the left hand side of the relation (15). To get a description for the right hand side of (15) we have to restore gauge invariance for the spin  $\ell$  field introducing the Goldstone spin  $\ell - 1$  field by shifting  $h^{(\ell)}$  with  $h^{(\ell)} + (a\nabla)\phi^{(\ell-1)}$ . This will affect only the mass term (48)

$$S_m[h^{(\ell)}, \phi^{(\ell-1)}] = \frac{m^2}{2} \int d^{d+1}z \sqrt{-g} (h_{\mu_1 \dots \mu_\ell}^{(\ell)} + \nabla_{(\mu_1} \phi_{\mu_2 \dots \mu_\ell)}^{(\ell-1)})^2. \quad (50)$$

The Stückelberg action  $S_{\mathcal{F}}^{GI}[h^{(\ell)}] + S_m[h^{(\ell)}, \phi^{(\ell-1)}]$  is gauge invariant under the gauge transformations (6) and  $\delta\phi^{(\ell-1)}(a; z) = -\epsilon^{(\ell-1)}(a; z)$  and describes the right hand side of (15). Moreover after integrating out the Goldstone field  $\phi^{(\ell-1)}$  in (50) we obtain according to the relation (47) the following part of the effective action

$$\begin{aligned} \frac{m^2}{2} \int d^{d+1}z_1 \sqrt{-g(z_1)} \int d^{d+1}z_2 \sqrt{-g(z_2)} \nabla \cdot h^{(\ell)}(z_1) K_G(z_1; z_2) \nabla \cdot h^{(\ell)}(z_2) \\ + \text{trace terms.} \end{aligned} \quad (51)$$

This dramatically simplifies the evaluation of the mass of the higher spin field generated by the one loop graph of section 1 with the  $h^{(\ell)}h^{(\ell-2)}\sigma$  interaction constructed in the previous section: It is enough to evaluate in the corresponding



loop function the coefficient in front of the term with the behaviour of the Goldstone mode propagator between two divergences of the external  $h^{(\ell)}$  fields (51). This will be done in the next section.

Another way to find the Goldstone mode is to construct directly transverse traceless states of  $h^{(\ell)}(z; a)$ . For doing that we introduce first so-called Lichnerowicz operator [13]. In  $AdS_{d+1}$  for a general rank  $\ell$  symmetric tensor this operator looks like

$$\mathbb{L}_{(\ell)} h^{(\ell)}(z; a) = \square h^{(\ell)}(z; a) - \frac{\ell(\ell + d - 1)}{L^2} h^{(\ell)}(z; a) + \frac{\ell(\ell - 1)a^2}{L^2} \bar{h}^{(\ell-2)}(z; a), \quad (52)$$

and obeys the following conditions

$$\frac{1}{\ell(\ell - 1)} \square_a \mathbb{L}_{(\ell)} h^{(\ell)}(z; a) = \mathbb{L}_{(\ell-2)} \bar{h}^{(\ell-2)}(z; a), \quad (53)$$

$$\frac{1}{\ell} \nabla^\mu \partial_{a^\mu} \mathbb{L}_{(\ell)} h^{(\ell)}(z; a) = \mathbb{L}_{(\ell-1)} \bar{h}^{(\ell-1)}(z; a), \quad (54)$$

$$(a \nabla) \mathbb{L}_{(\ell)} h^{(\ell)}(z; a) = \mathbb{L}_{(\ell+1)} (a \nabla) h^{(\ell)}(z; a). \quad (55)$$

This commutativity with the covariant derivatives and trace operation allows us to drop the degrees index of  $\mathbb{L}_{(\ell)}$  and consider it as a number in the calculation of the transverse traceless projection of  $h^{(\ell)}(z; a)$ . We can expand this projection in the following series

$$\begin{aligned} h^{(\ell)tt}(z; a) &= h^{(\ell)} + A(\mathbb{L})(a \nabla) \tilde{h}^{(\ell-1)} + B(\mathbb{L})(a \nabla)^2 \tilde{\tilde{h}}^{(\ell-2)} \\ &+ C(\mathbb{L})(a \nabla)^2 \bar{h}^{(\ell-2)} + O(a^2, (a \nabla)^3) + \dots \end{aligned} \quad (56)$$

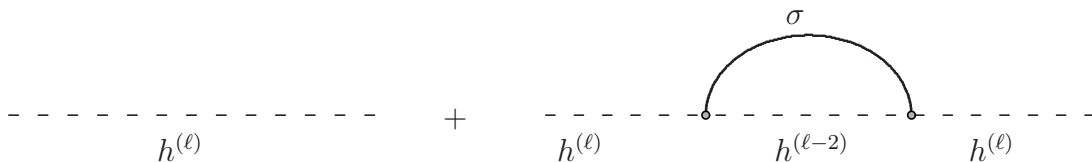
The tracelessness condition will express the next coefficients in terms of the previous ones but for us more interesting is the transversity condition  $\nabla^\mu \partial_{a^\mu} h^{(\ell)tt} = 0$  leading to the following solution for the coefficient  $A(\mathbb{L})$

$$A(\mathbb{L}) = \frac{-\ell}{\square + \frac{(\ell-1)(\ell+d-2)}{L^2}} = \frac{-\ell}{\mathbb{L}_{(\ell-1)}^t + \frac{2(\ell-1)(\ell+d-2)}{L^2}}, \quad (57)$$

here  $\mathbb{L}_{(\ell-1)}^t$  is the Lichnerowicz operator (52) for the rank  $\ell - 1$  traceless states. We see that the first pole in the  $tt$  projection behaves exactly as a propagator for a Goldstone tensor boson.

## 5 Propagator and loop graph

We study a perturbative expansion of the gauge field propagator



with the derivative action studied in section 3. To express these graphs we need the propagators for the fields  $h^{(\ell)}$ ,  $h^{(\ell-2)}$ , and  $\sigma$ . We use the results presented in [14, 15, 16]. The symmetric traceless tensor field  $h^{(\ell)}$  of rank  $\ell$  and dimension  $\Delta$  in  $AdS_{d+1}$  is massive in general according to the formula (14). This mass vanishes if

$$\Delta = d + \ell - 2. \quad (58)$$

In this case  $h^{(\ell)}$  allows gauge transformations as a symmetry. Representations  $[\Delta, \ell]$  of the universal covering group of the conformal group  $SO(d+1, 1)$  are unitary. From this point on we will use the Euclidian version of the  $AdS_{d+1}$  spaces and set to 1 the  $AdS$  radius  $L$ . Representations satisfying (58) are also called "exceptional". The propagator of  $h^{(\ell)}$  between two points  $z_1, z_2$  of  $AdS_{d+1}$  (symmetric traceless part) is a bitensor, it can be spanned by means of an algebraic bitensor basis  $I_1, I_2, I_3, I_4$  [14] by

$$\langle h^{(\ell)}(z_1) h^{(\ell)}(z_2) \rangle_{AdS} = \sum_{\substack{n_1, n_2, n_3, n_4 \\ n_1 + n_2 + 2n_3 + 2n_4 = \ell}} I_1^{n_1} I_2^{n_2} I_3^{n_3} I_4^{n_4} F_{n_1 n_2 n_3 n_4}^{[\Delta, \ell]}(\zeta(z_1, z_2)). \quad (59)$$

Here  $\zeta$  is the non-Euclidian cosine of the angle between  $z_1$  and  $z_2$

$$\zeta(z_1, z_2) = \cosh \Phi(z_1, z_2) = \frac{(z_1^0)^2 + (z_2^0)^2 + (\vec{z}_1 - \vec{z}_2)^2}{2z_1^0 z_2^0} \quad (60)$$

in Poincaré coordinates.

In the case of gauge fields  $h^{(\ell)}$  the propagators are most elegantly derived in the de Donder gauge by a recursive algorithm [16]. We neglect the trace terms, they can always be added at the end with simple arguments (see [14]). In order to derive (59) we start from

$$\Psi^{(\ell)}[F] = \sum_{k=0}^{\ell} I_1^{\ell-k} I_2^k F_k(\zeta) \quad (61)$$

and impose the field equation and the constraints of double tracelessness and de Donder gauge. We obtain [16]

$$F_0 = C \zeta^{-\Delta} {}_2F_1\left(\frac{1}{2}\Delta, \frac{1}{2}(\Delta+1); \Delta-\mu+1; \zeta^{-2}\right), \quad (62)$$

with  $\Delta$  (58) and  $\mu = \frac{1}{2}d$ ,

$$F_1(\zeta) = -\frac{\ell}{\zeta} F_0(\zeta), \quad (63)$$

$$F_k(\zeta) = (-1)^k \binom{\ell}{k} \zeta^{-k} F_0(\zeta) + f_k(\zeta), \quad (k \geq 2), \quad (64)$$

where  $f_k(\zeta)$  is asymptotically small

$$\frac{f_k(\zeta)}{F_k(\zeta)} = O\left(\frac{1}{\zeta}\right), \quad \zeta \rightarrow \infty. \quad (65)$$

Inserting these expressions in (61) we obtain

$$\Psi^{(\ell)}[F] = \left(I_1 - \frac{1}{\zeta}I_2\right)^\ell F_0(\zeta) + \sum_{k=2}^{\ell} I_1^{\ell-k} I_2^k f_k(\zeta). \quad (66)$$

In the limit  $\zeta \rightarrow \infty$  Dobrev's bulk-to-boundary propagator [18] can be derived from (66) (except the trace terms).

We set the normalization constant  $C$  equal to

$$C = \frac{\Gamma(\frac{1}{2}\Delta)\Gamma(\frac{1}{2}(\Delta+1))}{(4\pi)^{\mu+\frac{1}{2}}\Gamma(\Delta-\mu+1)}. \quad (67)$$

Using the fact that for  $\zeta \rightarrow 1$

$$\begin{aligned} & {}_2F_1\left(\frac{1}{2}\Delta, \frac{1}{2}(\Delta+1); \Delta-\mu+1; \zeta^{-2}\right) \\ &= \frac{\Gamma(\Delta-\mu+1)\Gamma(\mu-\frac{1}{2})}{\Gamma(\frac{1}{2}\Delta)\Gamma(\frac{1}{2}(\Delta+1))}(\zeta^2-1)^{-\mu+\frac{1}{2}} + O(1), \quad \Re\mu > \frac{1}{2}, \end{aligned} \quad (68)$$

and

$$\square \frac{\Gamma(\mu-\frac{1}{2})}{(4\pi)^{\mu+\frac{1}{2}}}(\zeta^2-1)^{-\mu+\frac{1}{2}} = -\delta(z_1, z_2) + \text{regular terms}, \quad (69)$$

we see that  $F_0(\zeta)$  appears as the kernel for the inverse wave operator  $(-\square + m^2)$  for the massive scalar field in Euclidian  $AdS_{d+1}$  space with  $m^2 = \Delta(\Delta-d)$ . In the next section we shall use this normalization (the same was used by D'Hoker, Freedman and coauthors [17] for the vector and graviton) for all symmetric traceless tensor fields be they massless or not.

Summarizing the consideration of this section we can formulate the following statements

a) The double traceless spin  $\ell$  Fronsdal propagator in the same fashion as the corresponding double traceless spin  $\ell$  current (see e.g. [8] and section 2) can be expressed as a combination of two traceless spin  $\ell$  and  $\ell-2$  propagators. The spin  $\ell-2$  propagator is connected with the Goldstone boson [16].

b) The leading term which is useful for our calculation in the next section, can be expanded as in (61) with the first and most important contribution  $F_0$  satisfying the massive scalar equation

$$[-\square + \Delta(\Delta-d)]F_0(\zeta(z_1, z_2)) = -\delta(z_1, z_2), \quad (70)$$

where  $\Delta$  is the dimension of the representation  $[\Delta, \ell]$  described by the initial gauge or massive field  $h^{(\ell)}$ .

The precise proof of the propagator structure is presented in [16].

## 6 The loop function

In the coordinate space the loop function is the product of the propagators for the fields  $h^{(\ell-2)}$  and  $\sigma$  (from now on  $d = 3$ )

$$\langle \sigma(z_1) \sigma(z_2) \rangle_{AdS_4} = \frac{1}{(4\pi\zeta)^2} K(\zeta), \quad K(\zeta) = \frac{\zeta^2}{\zeta^2 - 1} \quad (71)$$

We present the propagator of  $h^{(\ell-2)}$  by its  $F_0$  term and introduce in (37) a coupling constant which is yet unknown

$$A = \frac{g_\ell}{\sqrt{N}}. \quad (72)$$

Then the leading term of the loop function is

$$\frac{g_\ell^2}{N} \frac{\Gamma(\frac{1}{2}(\ell-1))\Gamma(\frac{1}{2}\ell)}{\Gamma(\ell-\frac{3}{2})(4\pi)^4} I_1^{\ell-2} \zeta^{-(\ell+1)} K(\zeta) {}_2F_1 \left[ \frac{1}{2}(\ell-1), \frac{1}{2}\ell; \ell - \frac{3}{2}; \zeta^{-2} \right]. \quad (73)$$

Note that in  $AdS$  space the product of two scalar propagators is not one scalar propagator but an infinite sum of scalar propagators with dimensions increasing

$$\Delta_{(n)} = \Delta_1 + \Delta_2 + 2n, \quad n \in \mathbb{N}_0. \quad (74)$$

As was described in the previous sections we have to use our interaction (37) and calculate the contribution of this loop function to the propagation of the longitudinal modes of  $h^{(\ell)}$ . Actually we have to extract the term  $\nabla \cdot h^{(\ell)} G_{[\ell+2, \ell-1]} \nabla \cdot h^{(\ell)}$ , where  $G_{[\ell+2, \ell-1]}$  is the Goldstone mode propagator (47) [12, 5] normalized in a proper way. For this we have to take the gradients of (73) in the following sense: For a bitensor  $\Psi^{(\ell)}$  such as (61) we introduce a map

$$\Psi^{(\ell)} \rightarrow \Psi^{(\ell+1)} = (a \cdot \nabla_1)(c \cdot \nabla_2) \Psi^{(\ell)}(\zeta; a, c) = \sum_{n=0}^{\ell+1} I_1^{\ell+1-n} I_2^n G_n(\zeta) + \text{trace terms}, \quad (75)$$

with

$$G_n = F_{n-1}''(\zeta) + (2n+1)F_n'(\zeta) + (n+1)^2 F_{n+1}. \quad (76)$$

For  $n = 0$  this gives

$$G_0(\zeta) = F_0'(\zeta) + F_1(\zeta). \quad (77)$$

$F_1$  is related to  $F_0$  by (63)

$$F_1 = -\frac{\ell-2}{\zeta} F_0. \quad (78)$$

Applying the gradient transformation to (73) we obtain then from (77)

$$-\frac{g_\ell^2}{N} \frac{\Gamma(\frac{1}{2}(\ell-1))\Gamma(\frac{1}{2}\ell)}{\Gamma(\ell-\frac{3}{2})(4\pi)^4} (2\ell-1) I_1^{\ell-1} \zeta^{-(\ell+2)} \left( 1 + O\left(\frac{1}{\zeta^2}\right) \right). \quad (79)$$

Next we have to identify in this expression the normalized Goldstone boson propagator of the representation  $[\ell + 2, \ell - 1]$ . For this purpose we use a normalizing factor analogous to (67) and get

$$\frac{\Gamma(\frac{1}{2}(\ell + 2))\Gamma(\frac{1}{2}(\ell + 3))}{\Gamma(\ell + \frac{3}{2})(4\pi)^2} I_1^{\ell-1} \zeta^{-(\ell+2)} {}_2F_1 \left[ \frac{1}{2}(\ell + 2), \frac{1}{2}(\ell + 3); \ell + \frac{3}{2}; \zeta^{-2} \right]. \quad (80)$$

Dividing (79) by (80) we obtain the mass squared of  $h^{(\ell)}$

$$\begin{aligned} m_\ell^2 &= \frac{g_\ell^2}{N} \frac{1}{(4\pi)^2} \frac{(2\ell - 1)\Gamma(\frac{1}{2}(\ell - 1))\Gamma(\frac{1}{2}\ell)\Gamma(\ell + \frac{3}{2})}{\Gamma(\frac{1}{2}(\ell + 2))\Gamma(\frac{1}{2}(\ell + 3))\Gamma(\ell - \frac{3}{2})} \\ &= \frac{g_\ell^2}{N} \frac{1}{(\pi)^2} \frac{(\ell - \frac{3}{2})(\ell - \frac{1}{2})^2(\ell + \frac{1}{2})}{(\ell - 1)\ell(\ell + 1)}. \end{aligned} \quad (81)$$

Remembering that the whole mechanism works only for  $\ell \geq 3$  we should multiply (81) with  $1 - \delta_{\ell 2}$ .

Comparing with the results obtained from the  $O(N)$  sigma model and  $AdS/CFT$  correspondence [6]

$$m_\ell^2 = \frac{16}{3N\pi^2}(\ell - 2) + O\left(\frac{1}{N^2}\right), \quad (82)$$

we can obtain the interaction constant  $g_\ell$

$$g_\ell^2 = \frac{16}{3} \frac{(\ell - 2)_4}{(\ell - \frac{1}{2})(\ell - \frac{3}{2})_3}. \quad (83)$$

An independent derivation of this coupling constant will be presented elsewhere.

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